

Math 347: Homework 8
Due on: Nov. 16, 2018

1. Let $n > k > j$. Prove that the greatest common divisor of $\binom{n}{k}$ and $\binom{n}{j}$ is not 1.
(Hint: Consider the Exercise 5) from Worksheet 10.)
2. (i) prove that 3 divides $4^n - 1$ for every $n \geq 1$ a natural number;
(ii) prove that 6 divides $n^3 + 5n$ for every positive integer n .
3. The *least common multiple* (lcm) of two natural numbers a and b is the smallest natural number divisible by both. Prove that

$$\gcd(a, b)\text{lcm}(a, b) = ab.$$

4. Prove using contradiction that the set of prime numbers is not finite.
5. Given $a, b, c \in \mathbb{Z}$, let $d = \gcd(a, b)$ and suppose that d divides c . Prove that the set of integer solutions to

$$ax + by = c$$

is nonempty. Express the set of all solutions in terms of a given solution and a, b and d .

(Hint: If you are stuck, consider the concrete problem of finding the integers solutions to

$$12x + 9y = 6.)$$